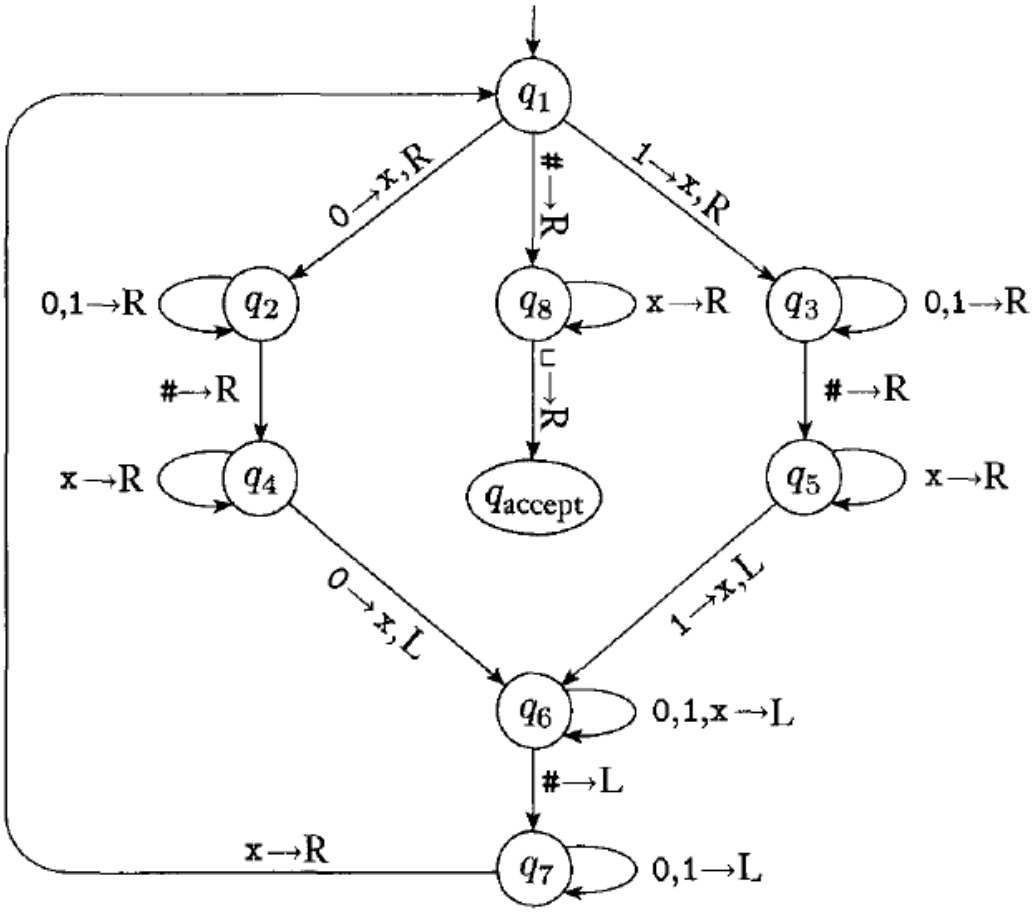
CPSC 326 Problem Set 3

Due April 18

1. Consider the following Turing machine which computes the language $\{w\#w \mid w \in \{0,1\}^*\}$:



What sequence of states does the machine go through when run on the following inputs?

a. 01#01
b. 10#11
c. 0000
d. #

2. Provide a complete Turing machine diagram for the language $\{w \mid w$ contains equal numbers of 0s and 1s$\}$.

3. Give a description of a Turing machines that decides the following language containing all strings in Hofstadter's PQ system. Strings contain X hyphen characters, followed by a 'p', followed by Y hyphen characters followed by a 'q', followed by $X+Y$ hyphen characters. For instance the string "- - p - - - q - - - - -" should be accepted, but the string "- - p - - q - - - - -" should be rejected. You do not need to provide a complete diagram, but the description should be detailed enough that we can visualize what the machine is doing.

4. A "Unidirectional Turing machine" is one that doesn't move the tape head left or right. Instead it can either move the tape head right, or it can choose to keep the tape head in the same location. Is this type of machine equivalent to a standard Turing machine? Justify your answer.

5. A "Type-writer Turing machine" is one that doesn't move the tape head left or right. Instead it can either move the tape head right, or it can move the tape head all the way back to the leftmost position (like an old-fashioned typewriter). Is this type of machine equivalent to a standard Turing machine? Justify your answer.

6. ALL_{DFA} is the problem of deciding whether a given DFA accepts all strings. Show that ALL_{DFA} is decidable.

7. On the other hand, ALL_{CFG}, which is the problem of deciding whether a given CFG produces all strings, is *not* decidable. Use this result to show that EQ_{CFG} is also undecidable. EQ_{CFG} is the problem of deciding whether two CFGs are equivalent.